SpaceEx:

Scalable Verification of Hybrid Systems

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CAV
Snowbird, Utah, 18 July 2011

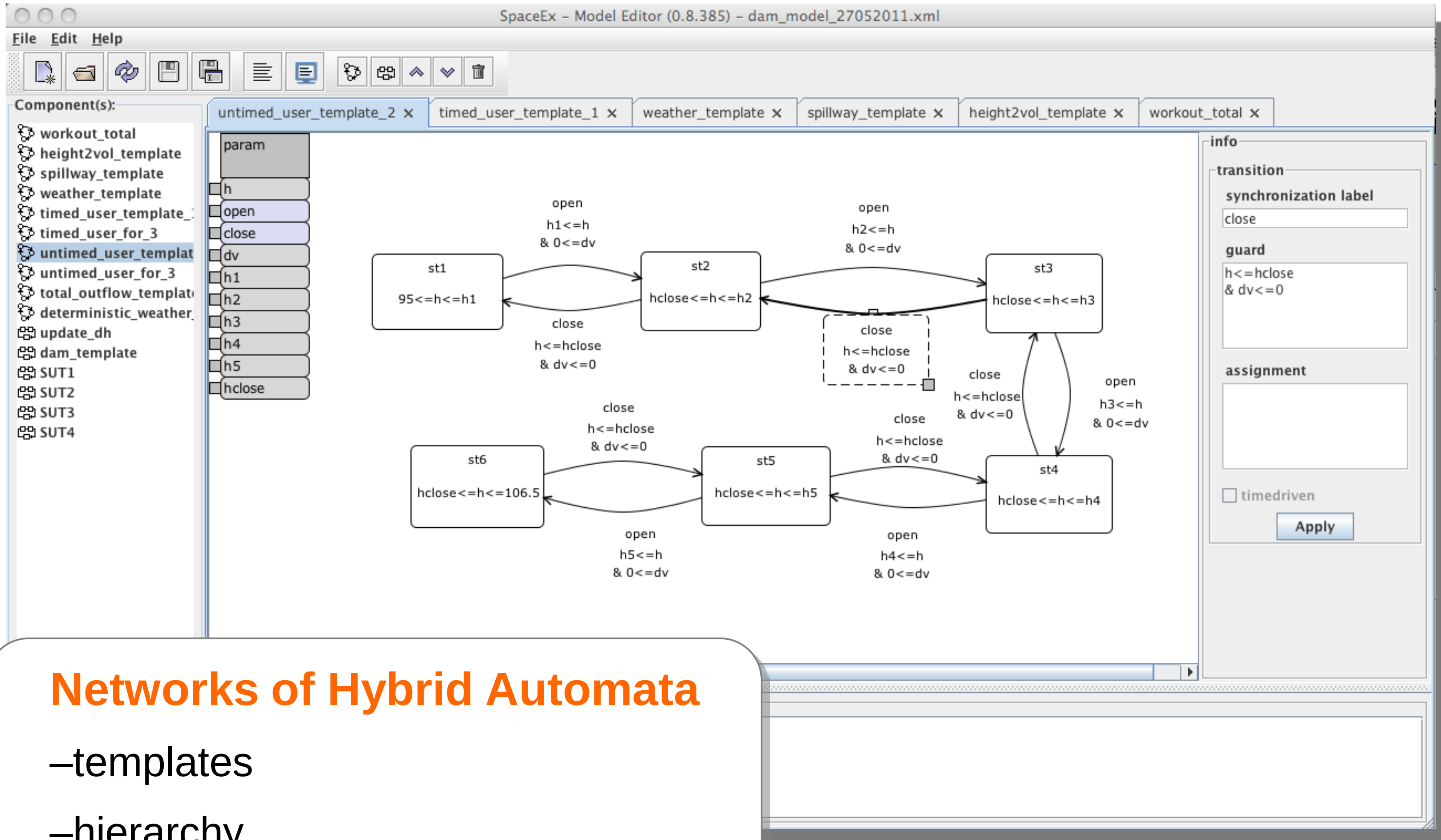
Outline

- **SpaceEx Verification Platform**
- **Hybrid Systems Reachability**
- **SpaceEx Reachability Algorithm**
  - Time Elapse with Support Functions
  - Transition Successors Combining Support Functions and Polyhedra
  - Fixpoint Algorithm: Clustering
- **Examples**

SpaceEx Verification Platform

- **Platform for developing verification algorithms**
  - Analysis Core (90kloc C++)
  - Model Editor
  - Web Interface
- **Provides data structures, operators, infrastructure**
  - proprietary polyhedra library
  - number type is templated (substitute your own)
  - interfaces to linear programming solvers (GLPK,PPL), Parma Polyhedra Library, ode solvers (CVODES)
- **Open Source: spaceex.imag.fr**

SpaceEx Model Editor

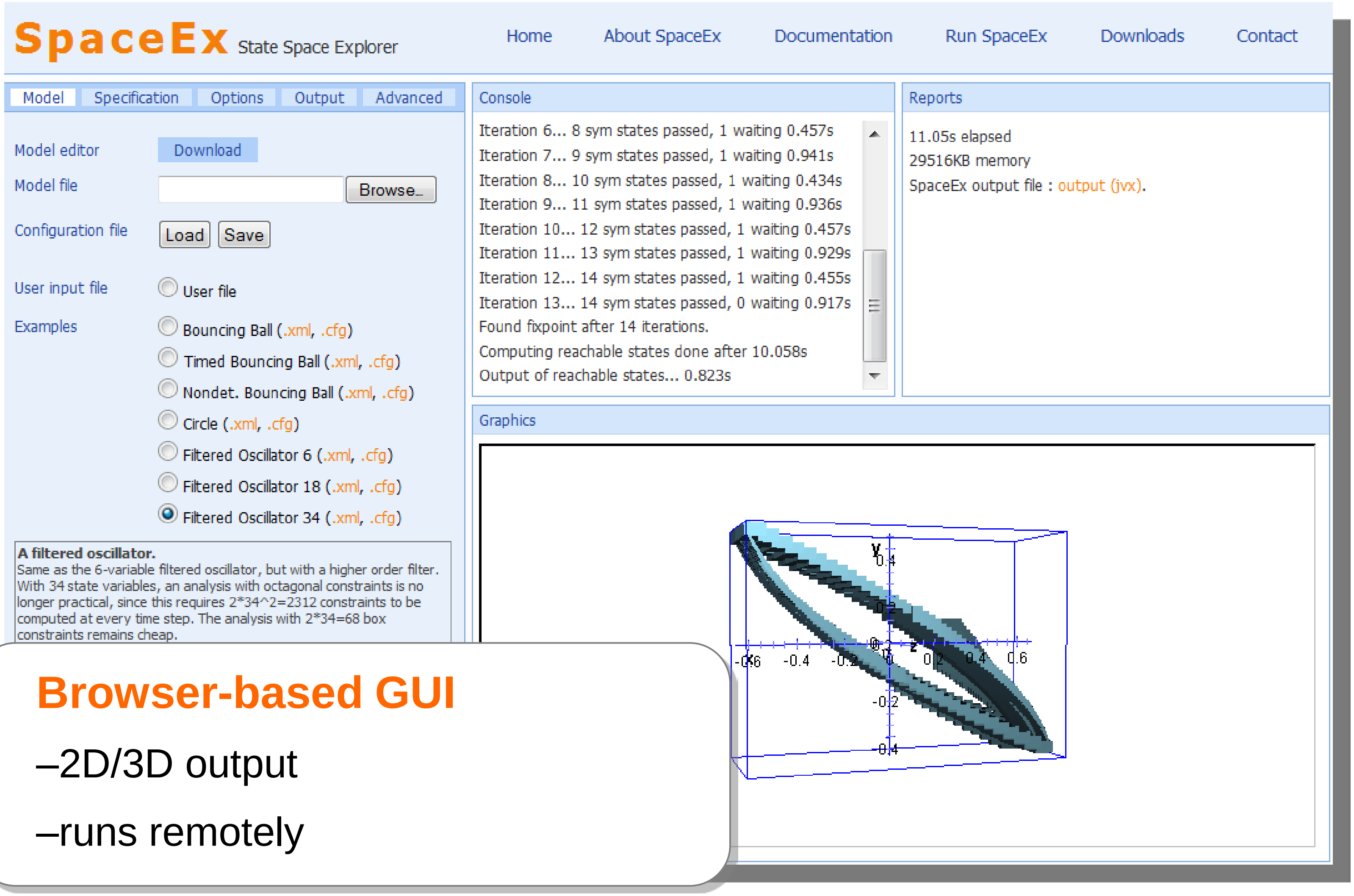


Networks of Hybrid Automata

–templates

–hierarchy

SpaceEx Web Interface

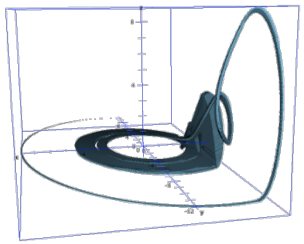


Browser-based GUI

–2D/3D output

–runs remotely

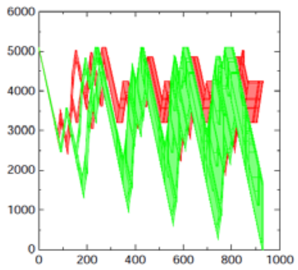
SpaceEx Reachability Algorithms



Support Function Algo

–many continuous variables

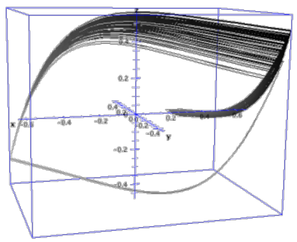
–low discrete complexity



PHAVer

–constant dynamics (LHA)

–formally sound and exact

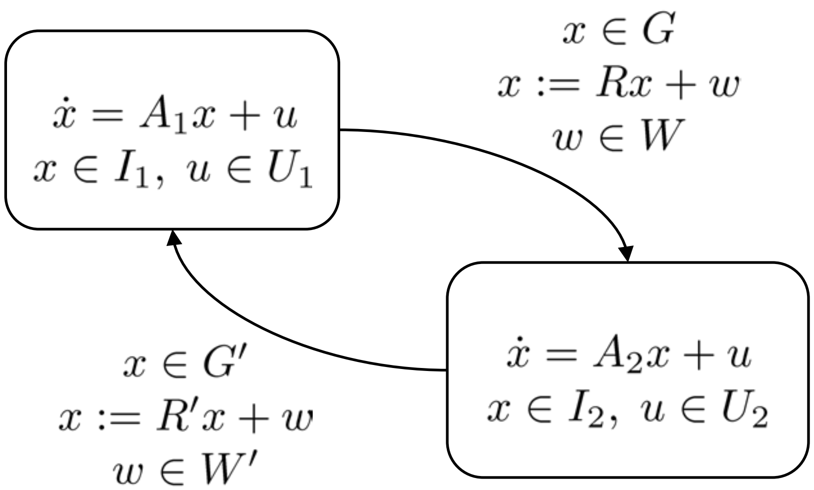


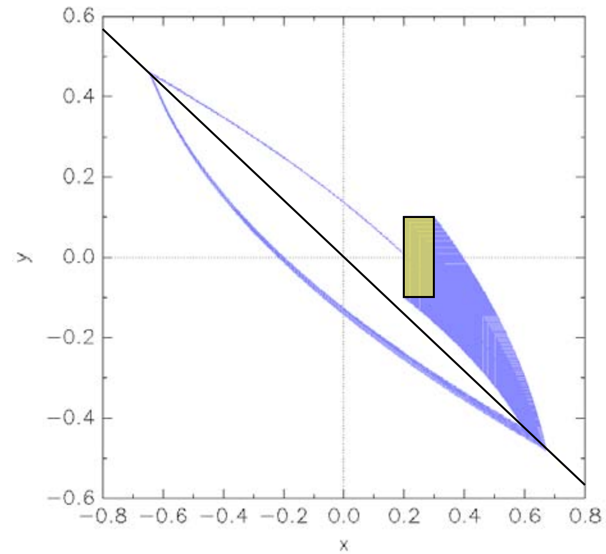
Simulation

–nonlinear dynamics

–based on CVODE

Hybrid Automata with Affine Dynamics





- **linear differential equations**
- **can be highly nondeterministic:**
  - additive “inputs” u,w model continuous disturbances (noise etc.)
  - uncertain switching regions
  - uncertain switch result

Reachability of Hybrid Automata

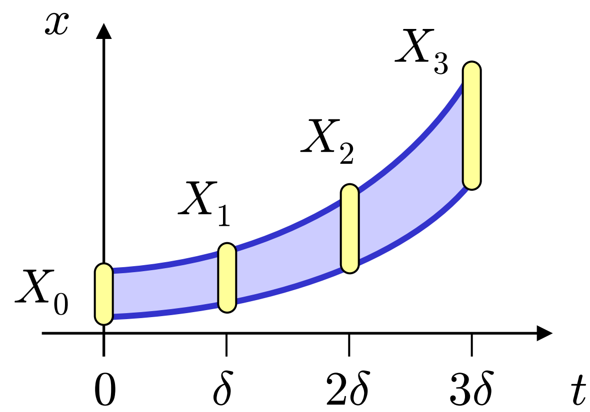
- **reachability is hard for continuous dynamics**
  - complex, nonconvex sets
- **even harder for hybrid dynamics**
  - involves reachability of continuous dynamics
  - plus event detection over a dense domain
- **approximations needed that are efficient but accurate for large number of variables**

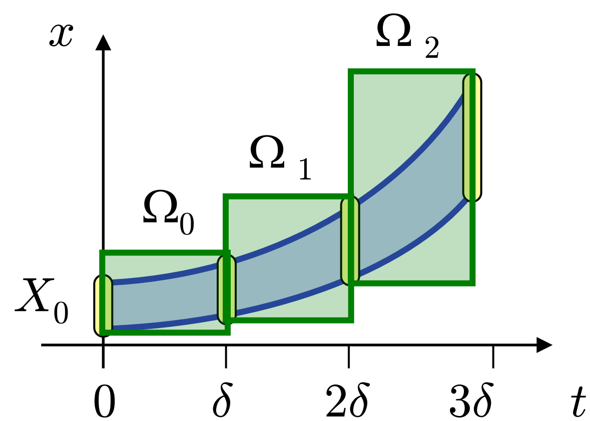
Improving Reachability with Support Functions

- **Efficient time elapse algorithm for high dimensions**
  - Le Guernic, Girard, CAV 2009
- **Problems & proposed improvements**
  - set representation inefficient for reachability fixpoint algorithm (intersection & containment)
    - **switch to polyhedra and back when better**
  - large, uniform overapproximation with conservative error bounds
    - **more accurate, non-uniform overapproximation**
  - fixed time step
    - **adaptive, multi-scale time step**

Reachability of Affine Continuous Dynamics

$$x(t) = \underbrace{e^{A\delta}x(0)}_{\text{autonomous dynamics}} + \underbrace{\int_0^{\tau} e^{A(\delta-\tau)}u(\tau)d\tau}_{\text{influence of inputs}}$$





- **solution at discrete time steps**
- **cover flowpipe with convex sets Ω_i: approximation model**

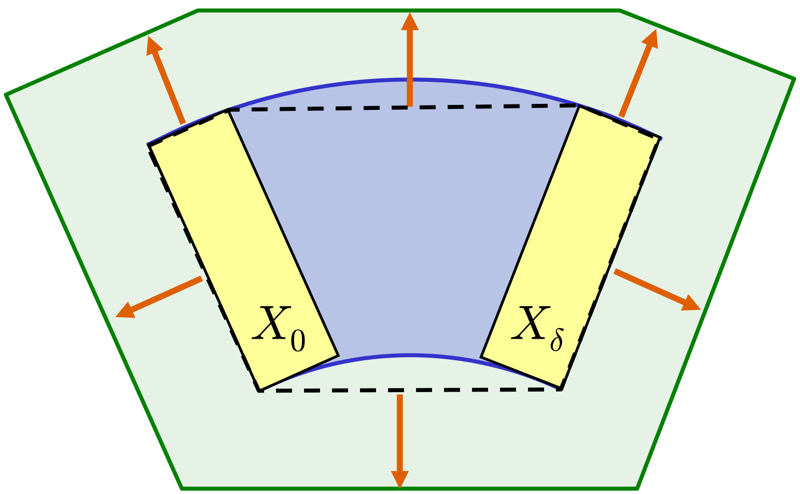
Approximation Models – Prev. Work

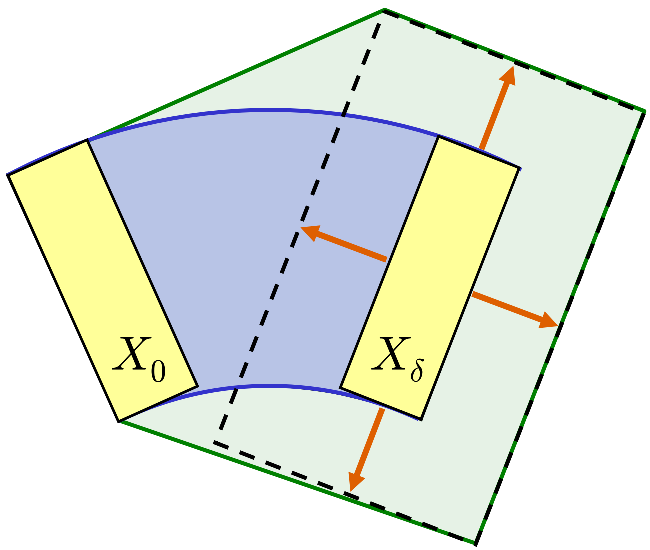
- **convex hull constraints + bloat with $\sim e^{\|A\|\delta}$**

  Asarin, Dang et al., HSCC 2000

- **bloat last set with $\sim e^{\|A\|\delta}$ + convex hull**

  Le Guernic, Girard, CAV 2009



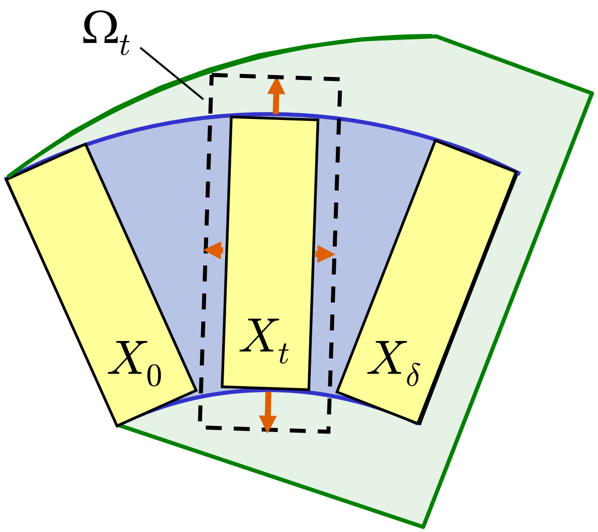


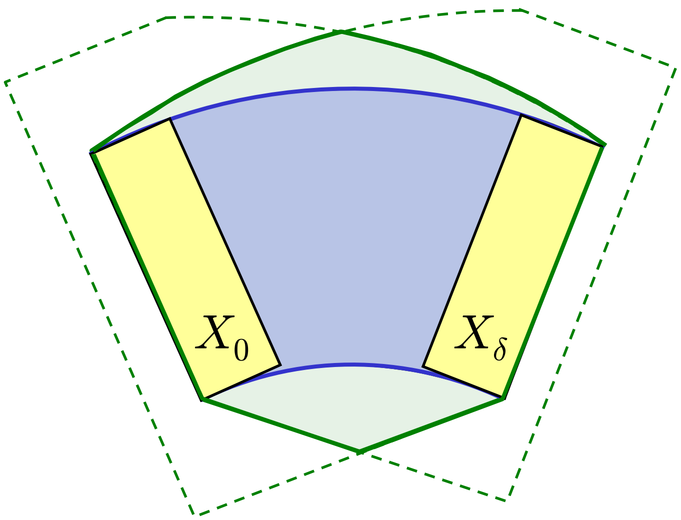
- **error large and uniform**
- **exponential cost**

- **error large and uniform**
- **efficient for high dimensions**

New Approximation Model

- approximate set for each t + bloat with $\sim e^{\text{abs}(A)\delta} A X_0$
- intersect forward and backward approximations

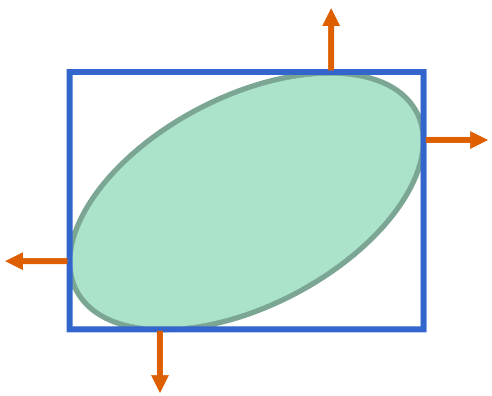




- error small and non-uniform thanks to math tricks
- without inputs: exact at t=0 and t=δ

Representing of Convex Sets

- **Approximation with Supporting Halfspaces**
  - given template directions = **outer polyhedral approximation**

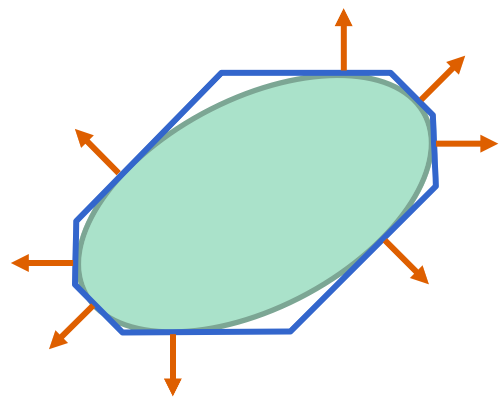


axis ($\pm x_i$)

$\Downarrow$

bounding box

2n facets

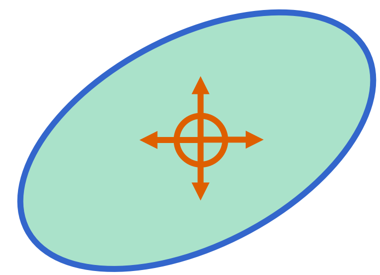


octagonal ($\pm x_i \pm x_j$)

$\Downarrow$

bounding polytope

$2n^2$ facets



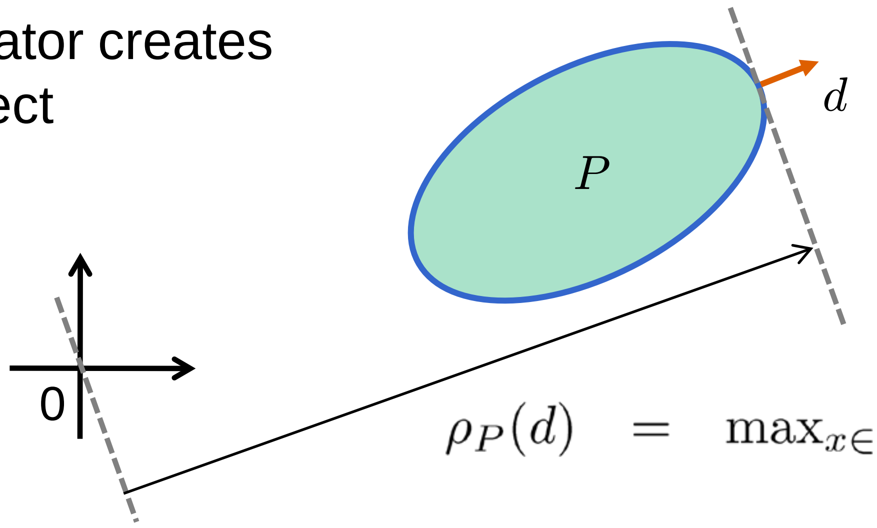
all directions

$\Downarrow$

exact set

Representation of Convex Sets

- **Support Function**
  - direction $\rightarrow$ position of supporting halfspace
  - exact set representation
- **Implemented as function objects**
  - applying an operator creates new function object



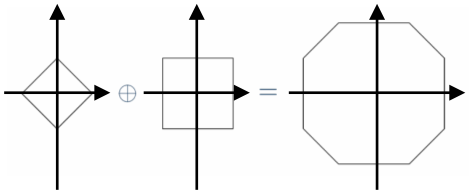
$$\rho_P(d) = \max_{x \in P} d^T x$$

New Approximation Model

- **Choose set representation on operator efficiency**

	Polyhedra			
Operators	Constraints	Vertices	**Zonotopes**	**Support F.**
Convex hull	--	+	-	++
Linear transform	+/-	++	++	++
Minkowski sum	--	--	++	++

Minkowski sum:

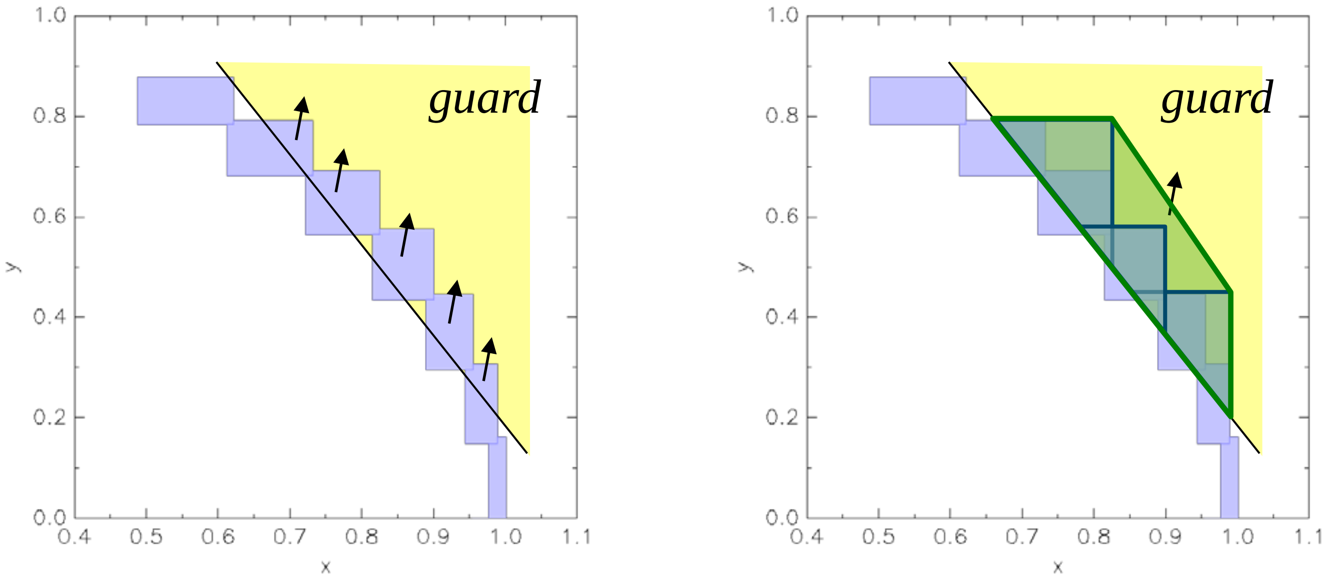


New Approximation Model

- efficiently computable with support functions
- set computation reduced to set of scalar optimization problems
- error bounds for each template direction

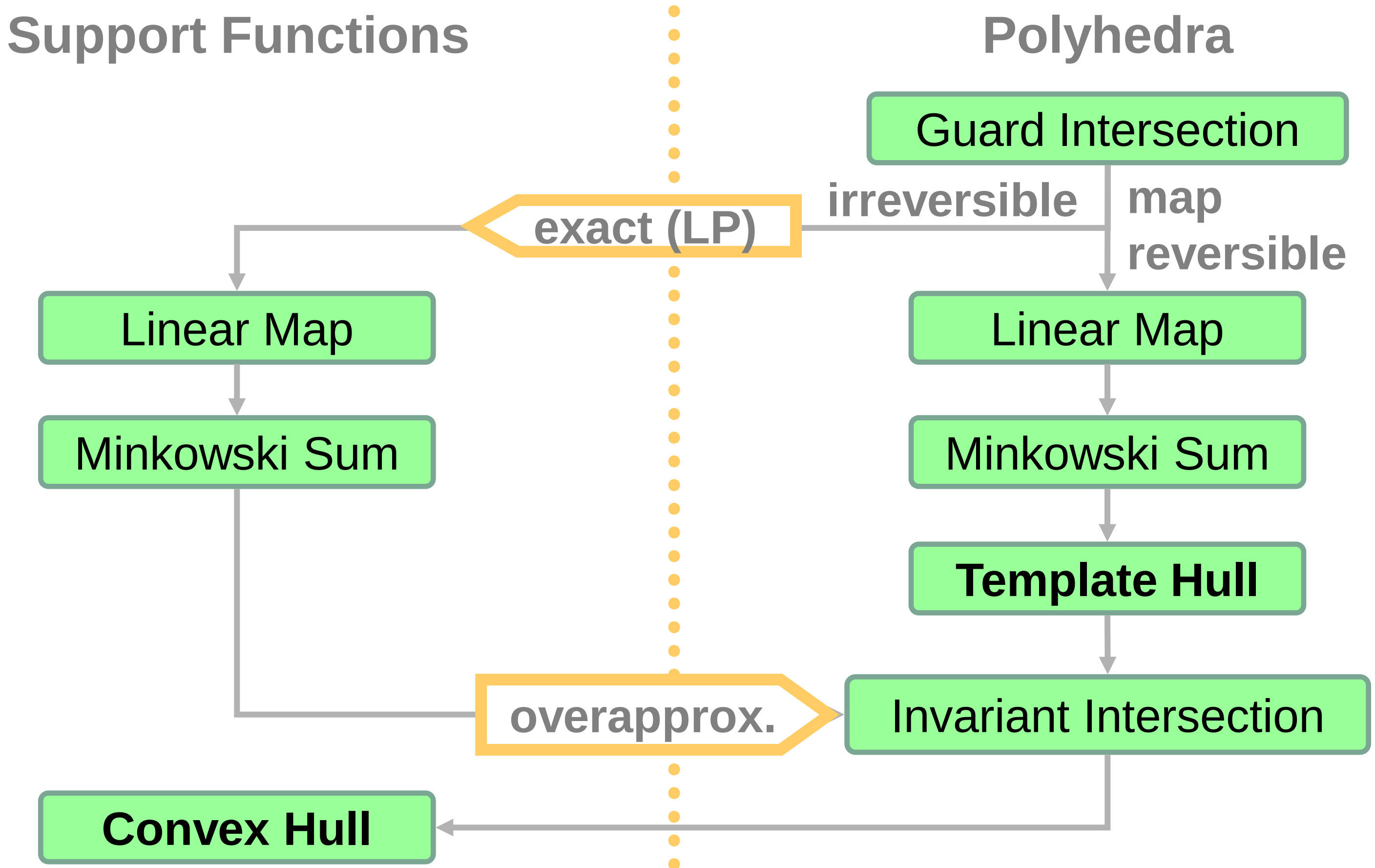
- but: intersecting with invariant inefficient for s.f.
  - switch to outer polyhedron approx.

Clustering



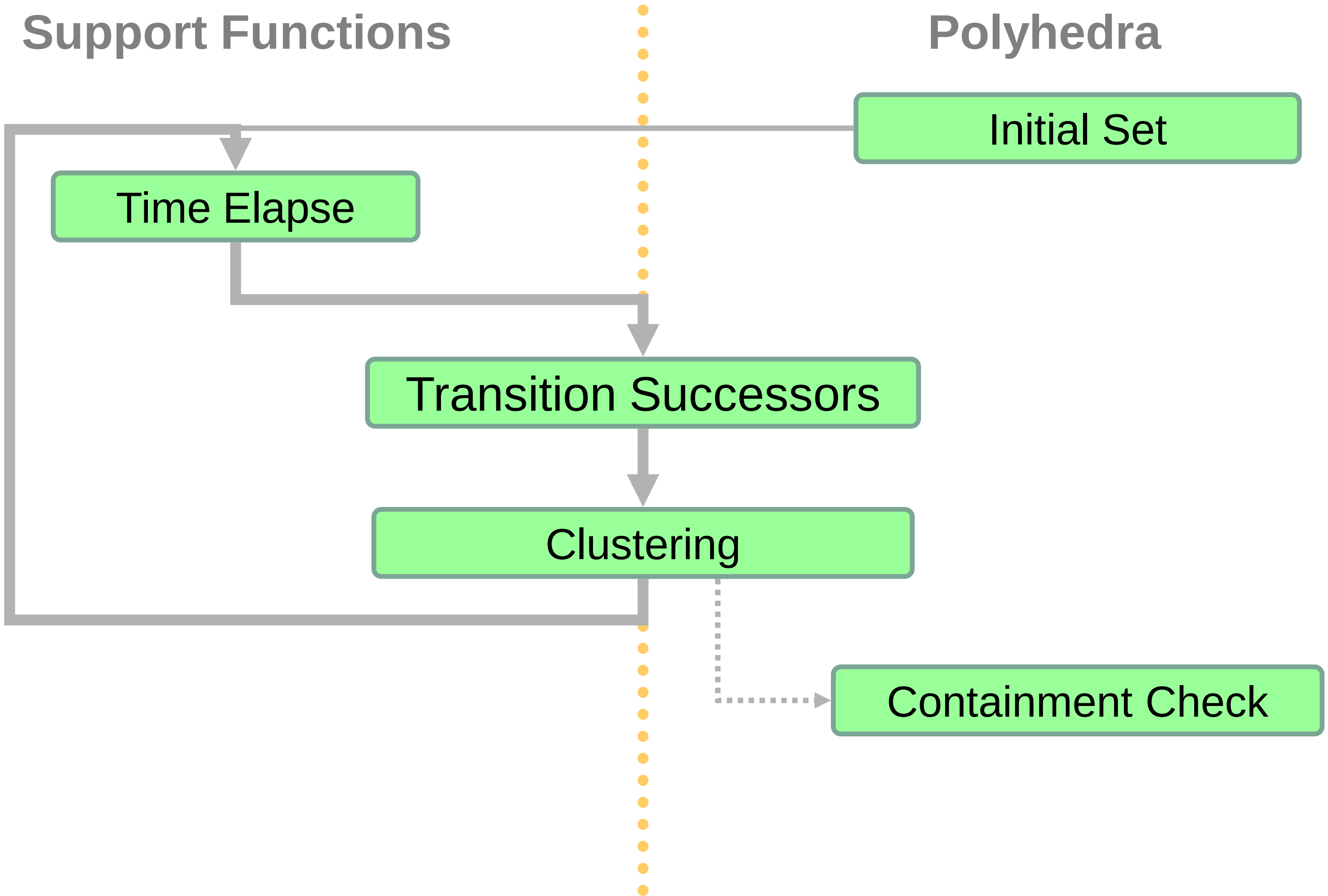
- **Every convex set spawns a new flowpipe**
  - number of sets explodes
- **Cluster using template hull (outer polyhedron) and convex hull**

Transition Successors with Clustering



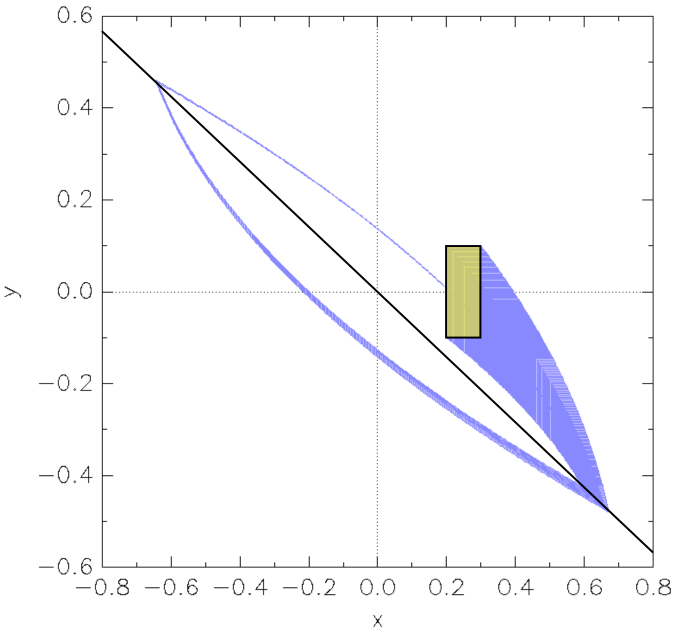
after intersection because
contained in convex invariant

Fixpoint Algorithm



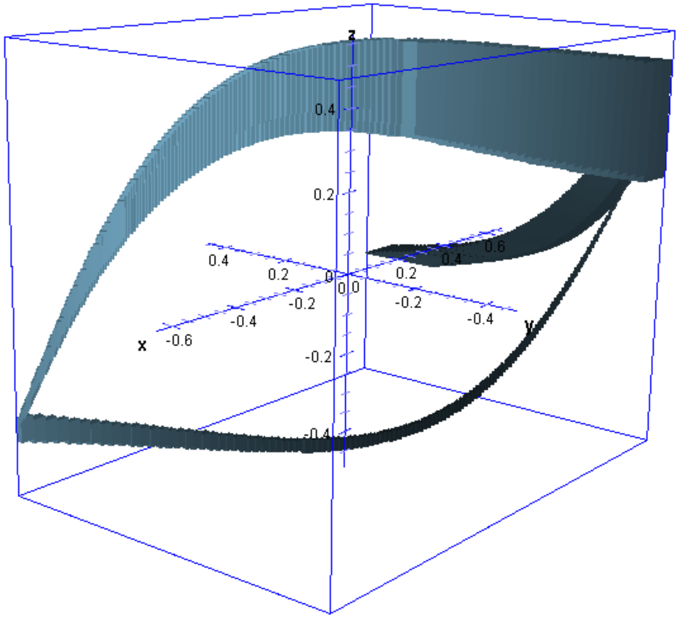
Example 1:
Filtered Switched Oscillator

- **Switched oscillator**
  - 2 continuous variables
  - 4 discrete states
  - similar to many circuits (Buck converters,…)
- **plus linear filter**
  - m continuous variables
  - dampens output signal
- **affine dynamics**
  - total $2 + m$ continuous variables

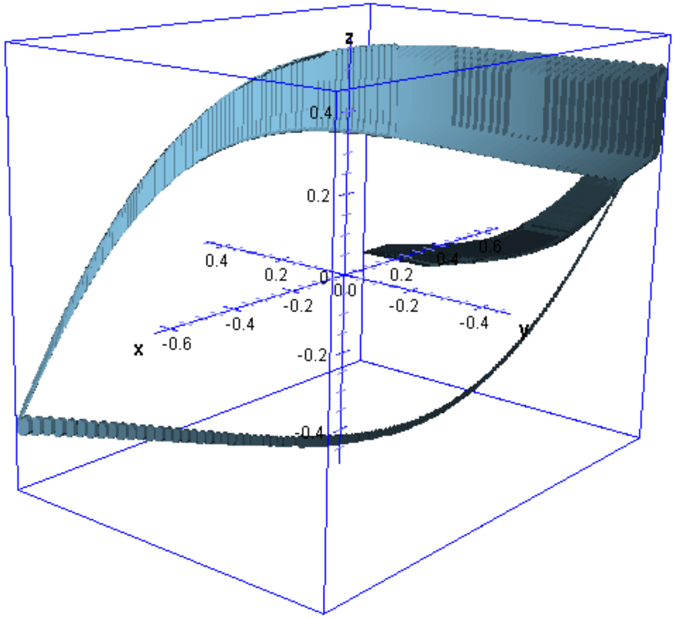


Filtered Switched Oscillator

- **Low number of directions sufficient?**
  - here: 6 state variables



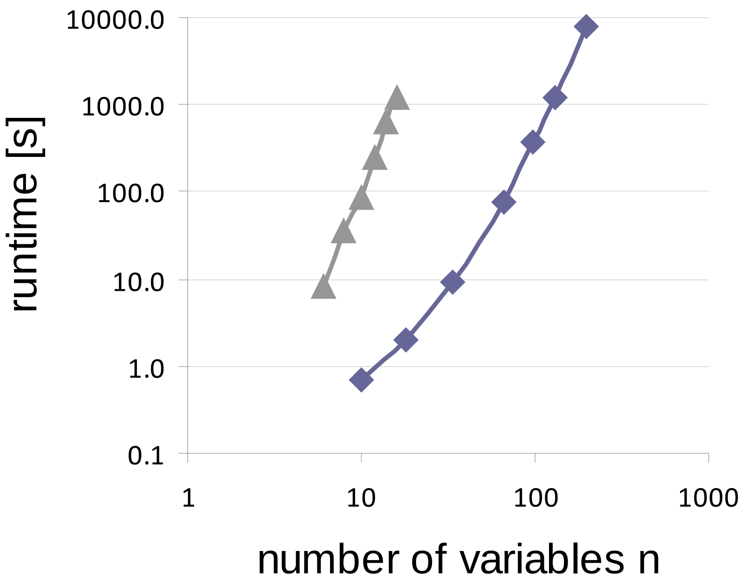
12 box constraints
(axis directions)



72 octagonal constraints
($\pm x_i \pm x_j$)

Example 1: Filtered Switched Oscillator

- **Scalable:**
  - fixpoint reached in $O(nm^2)$ time
  - box constraints: $O(n^3)$
  - octagonal constraints: $O(n^5)$



- **Clustering indispensible**
  - 57 sets take first jump
  - combination of template and convex hull: compromise in speed and accuracy

Example 2: Controlled Helicopter

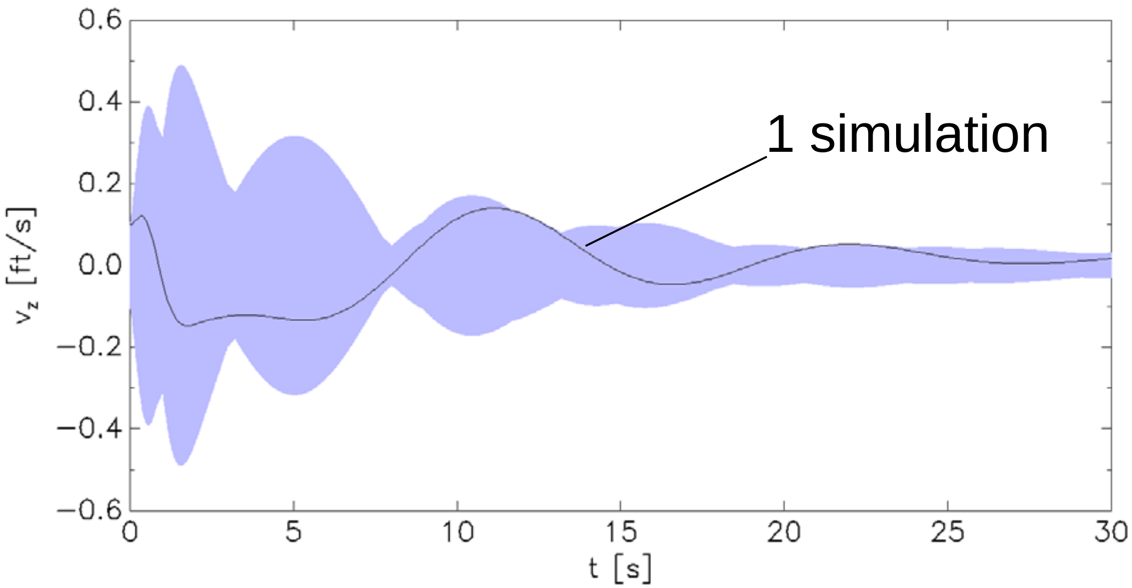
- **28-dim model of a Westland Lynx helicopter**
  - 8-dim model of flight dynamics
  - 20-dim continuous $H\infty$ controller for disturbance rejection
  - stiff, highly coupled dynamics

S. Skogestad and I. Postlethwaite, Multivariable Feedback Control: Analysis and Design. John Wiley & Sons, 2005.

Example 2: Controlled Helicopter

- **Reachability for uncertain initial states:**
  - old approx.: 200s error large
  - new approx.: 24s error < 0.025
  - variable time step: 14s error < 0.025
    (without interpolation)

vertical speed



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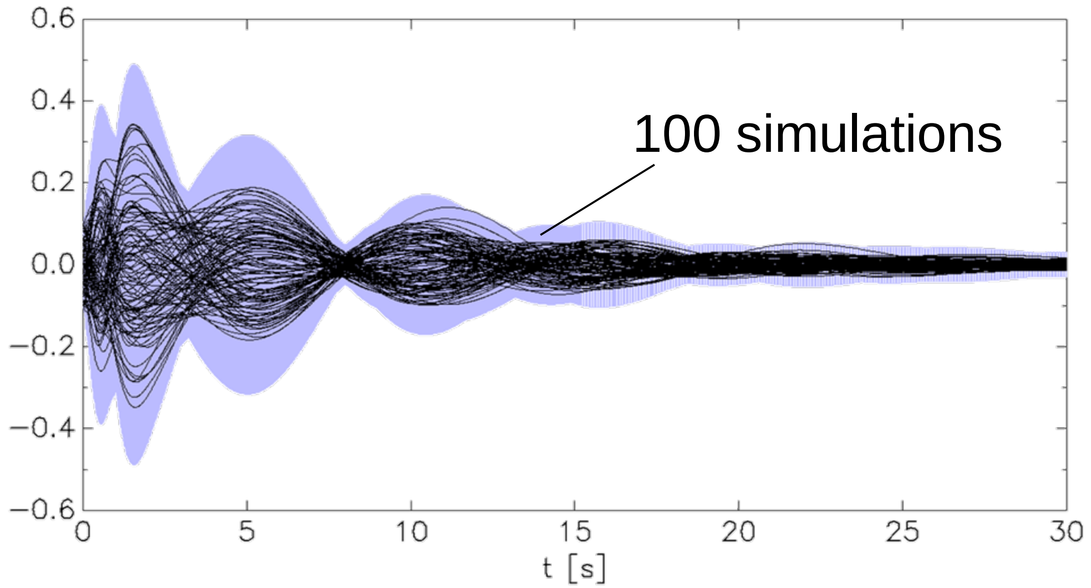

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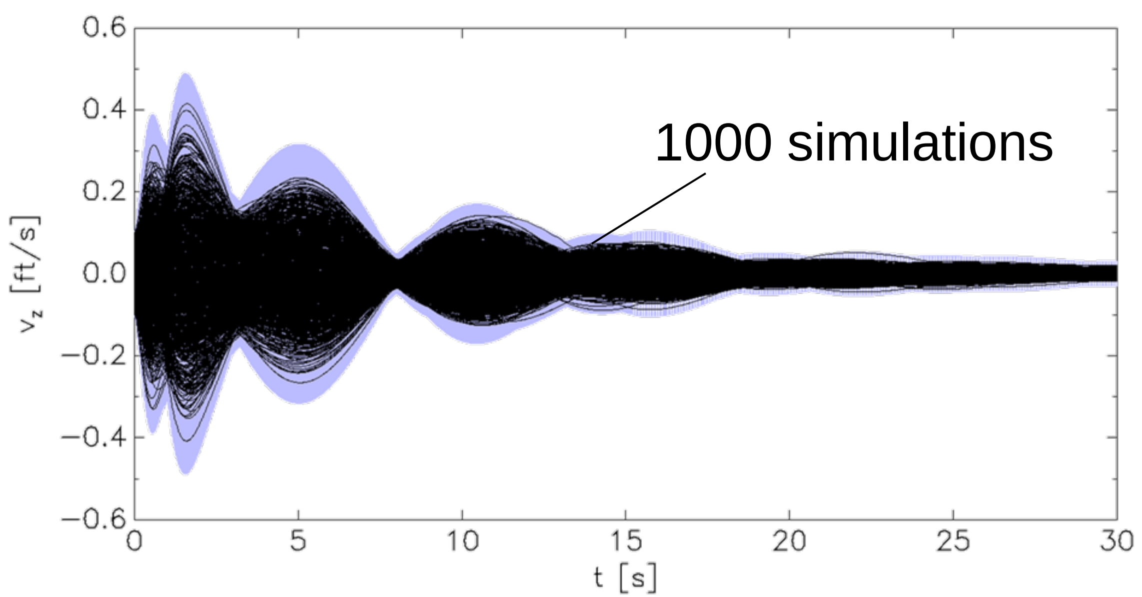

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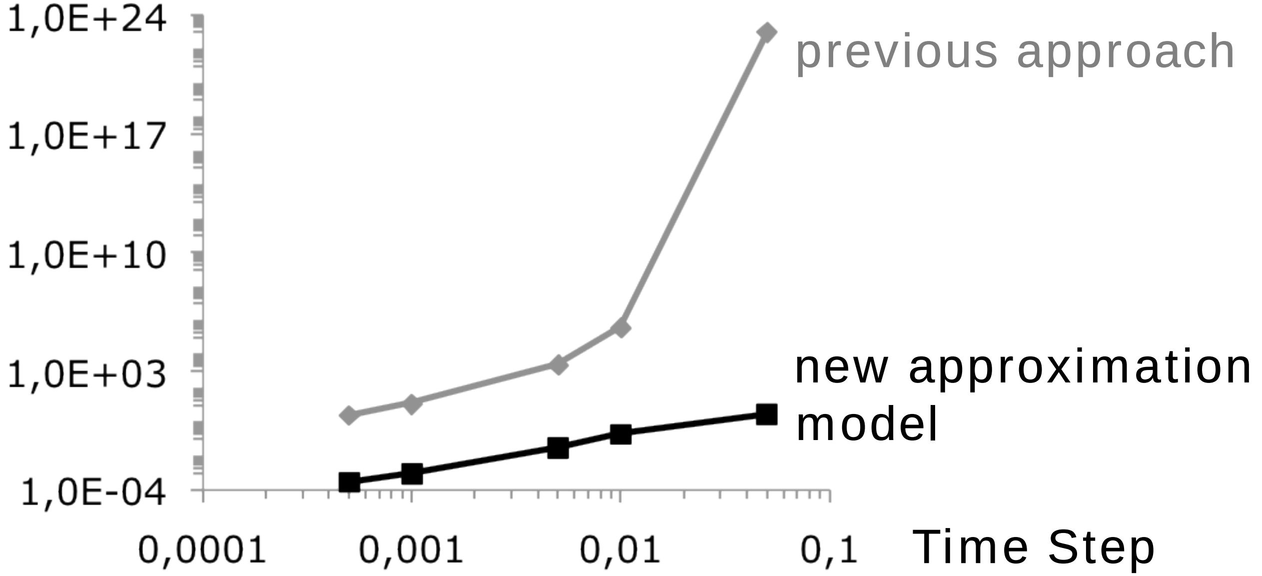
Example 2: Controlled Helicopter

- **Max error per template direction:**



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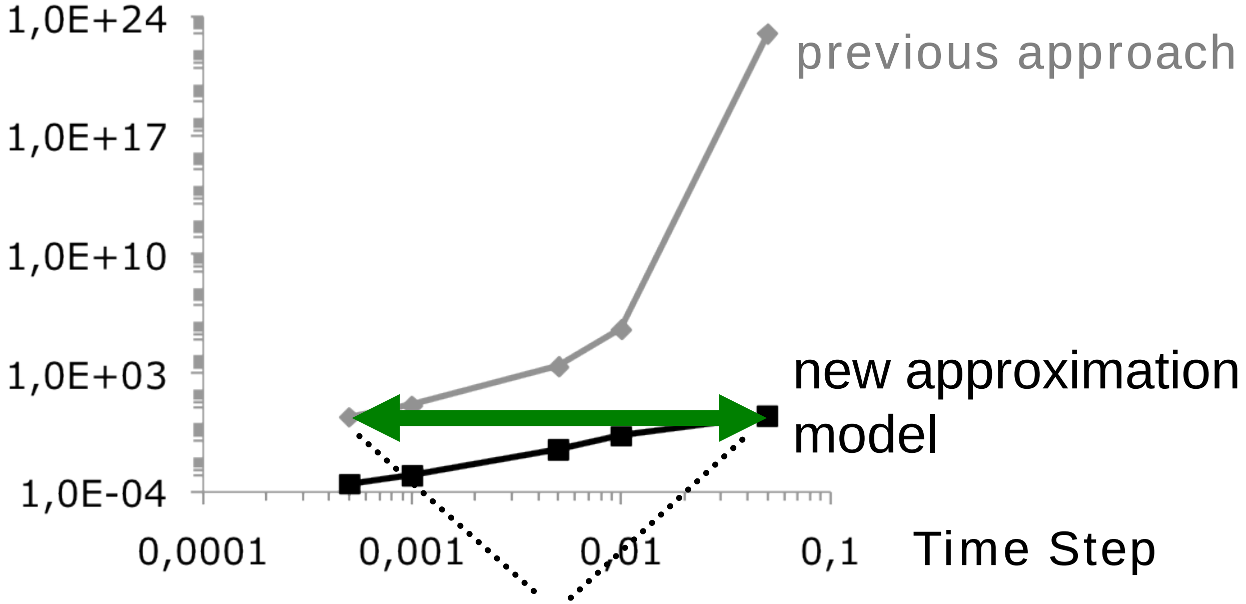

100x bigger time step for same error

Conclusions

- **SpaceEx Verification Platform**
  - available at **spaceex.imag.fr**
  - tutorial with solutions for course work
- **Scalable reachability for piecewise affine dynamics**
  - fixpoint computation with 200+ variables
- **Algorithmic improvements**
  - approximation improved significantly
  - switching set representations for best efficiency
  - variable time step with error bounds